ANOMALOUS PROPAGATION, LOSS AND RADIATION EFFECTS IN OPEN WAVEGUIDES WITH GYROTROPIC MEDIA

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Abstract

A complete investigation of the electromagnetic behavioral features is presented for a class of open waveguides employing gyrotropic substrates, which find applications in various microwave and millimeter-wave devices. Referring to a representative nonreciprocal open structure with transversely-magnetized ferrite, a rigorous analysis is led first to derive the dispersion properties for the modal spectrum in the lossless case, emphasizing particular propagation properties. Losses in the anisotropic medium are then taken into account, and the consequent important modifications are illustrated. Moreover, different kinds of complex solutions are studied as concerns their physical nature. Interesting practical implications are thus derived by evaluating, in connection with proper sources, the effective contributions of such complex waves to radiation phenomena.

1. Introduction

Many important applications in the microwave area make use of a variety of anisotropic and nonreciprocal structures based on gyrotropic media [1]. An outstanding type of materials for this class of devices is represented by ferrites, employed since many years in the design of various components (circulators, isolators, phase shifters, modulators, switches, etc.) [2,3].

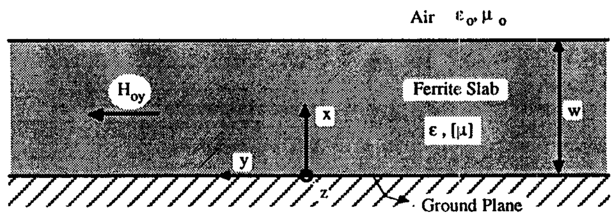
A renewed interest in such devices has been addressed to investigate ever more sophisticated behavioral features [4,5]. On this ground, the present work deals with a complete electromagnetic analysis of a fundamental class of ferrite waveguides of open type. Several disregarded propagation (Sect. 2) and radiation (Sect. 3) phenomena have been investigated and discussed.

2. Propagation features in lossless and lossy structures

2.1. Analysis of waves in open guides derived from a grounded ferrite slab

The basic kind of structure we are interested in represents nonreciprocal open waveguides with a transversely-magnetized ferrite slab, placed on a grounded metallic plane: the geometry and the characteristic parameters are shown in Fig. 1. We remind also that the behavior of such a structure is practically equivalent to that one of a bisected nonradiative dielectric (NRD) guide [5].

It is known that for such kind of structures, when the independence of the fields is assumed along the static magnetization direction, anisotropic effects can be found for TE modes only [1,5]. This is the case extensively analyzed here.



$$[\mu] = \mu_o[\mu_r] = \mu_o \begin{bmatrix} \mu_1 & 0 & -j\mu_2 \\ 0 & 1 & 0 \\ j\mu_2 & 0 & \mu_1 \end{bmatrix}, \quad \begin{cases} \mu_1 = 1 + \dfrac{\omega_m \omega_o}{\omega_o^2 - \omega^2} \\ \mu_2 = \dfrac{\omega\, \omega_m}{\omega_o^2 - \omega^2} \end{cases}, \quad \mu_e = (\mu_1^2 - \mu_2^2)/\mu_1, \quad \omega_o = -\gamma H_o, \quad \omega_m = -\frac{\gamma}{\mu_o} M_o$$

γ, gyromagnetic ratio

Fig. 1 - The class of anisotropic waveguides under investigation is represented by a grounded transversely-magnetized ferrite slab, here shown with the relevant parameters. Anisotropic effects are present in TE waves, that do not depend on the bias direction. In the simulations, this parameter choice has generally been fixed: $\varepsilon_r = 10$; $w = 6$ mm; $M_{oy} = 0.3$ Wb/m^2; $H_{oy} = (1/4\pi)\ 10^6$ A/m.

Introducing an original and straightforward equivalent transmission-line formalism for anisotropic structures [6], it has been possible to immediately achieve the dispersion equation for the grounded-slab TE modes, from which accurate numerical solutions have been derived:

$$\begin{cases} \sqrt{k_e^2 - k_z^2}\cot\left(\sqrt{k_e^2 - k_z^2}\, w\right) + \mu_e \sqrt{k_z^2 - k_o^2} + k_z \dfrac{\mu_2}{\mu_1} = 0 \\ k_o = \omega\sqrt{\mu_o \varepsilon_o},\ k_e^2 = k_o^2 \varepsilon_r \mu_e,\ k_{xf} = \sqrt{k_e^2 - k_z^2},\ k_{xo} = \sqrt{k_o^2 - k_z^2} \end{cases} \quad (1)$$

where $k_z = \beta_z - j\alpha_z$ is the longitudinal wavenumber, k_{xf} is the transverse wavenumber in the ferrite (in the air a decaying field having $k_{xo} = -j\alpha_{xo}$ is assumed for guided modes), and an 'effective' ferrite wavenumber k_e is also introduced.

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Various interesting propagation phenomena have been found from the evaluation of the modal wavenumbers. In fact, this analysis has allowed us to emphasize different types of both real (Sect. 2.2) and complex solutions (Sect. 2.3). The deep modifications of the ideal behaviors due to material dissipation effects are then considered (Sect. 2.4).

2.2. *Guided modes in the lossless structure*

In order to have a background on the general features of our structure, the lossless case is considered first, by evaluating all the modal wavenumbers. Here we point out briefly only the main characteristics of proper real modes derivable from the dispersion behaviors (normalized phase constant β_z/k_o vs. f).

The typical full-wave spectrum of guided modes, for a fairly common choice of the parameters (see Fig. 1), is shown in Fig. 2. As is known [5], three basic frequency ranges are generally present (related with the dispersive characteristics of the permeability tensor): zone 1) $f<f_1$; zone 2) $f_1<f<f_2$; zone 3) $f>f_2$.

At f_1 (where $\mu_1=0$) and at f_2 (where $\mu_e=0$), vertical asymptotes are present for β_z/k_o. Both in zone 1 and 3, horizontally-stationary field configurations usually exist (real k_{xf}), while in zone 2 (where $\mu_e<0$) there are horizontally-evanescent field configurations (imaginary k_{xf}).

Such modes are classified as $TE_{n\pm}^{(1)}$, $TE_{n\pm}^{(2)}$, $TE_{n\pm}^{(3)}$, where the apex is related to the region, the $\pm$ sign concerns either 'direct' (positive β_z) or 'reverse' (negative β_z) propagation, and n is an odd integer that orders in each zone the eigenvalues of the grounded slab.

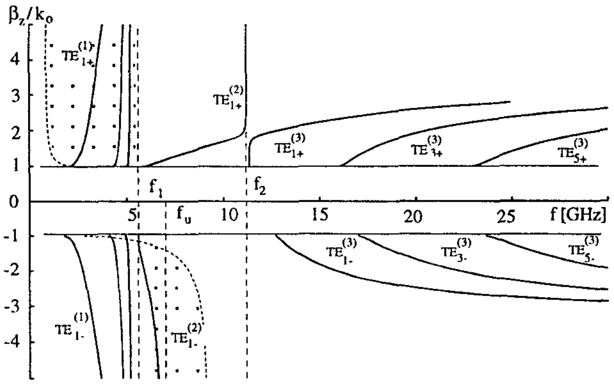

Fig. 2 - Full-wave nonreciprocal dispersion behaviors β_z/k_o vs. f for TE guided modes in grounded ferrite slab. Presence of zones 1, 2, and 3, separated by asymptotes at $f=f_1$ and $f=f_2$. Curved dashed lines limit the zones (dotted areas) where power is carried in opposite directions in the ferrite and in the air.

Some general considerations are briefly emphasized on the modal characteristics. It has been noticed that inside region 2 only a guided solution is found, i.e., $n=\pm 1$, and a 'unidirectional' frequency range exists, between f_u and f_2, in which only one direct wave ($\beta_z/k_o>0$) and no reverse wave ($\beta_z/k_o<0$) can propagate [5].

On the contrary, the number of guided modes in regions 1 and 3 is unlimited (only the first three curves of the infinite set of modes are represented in Fig. 2). Anyway, since zone 1 has a finite frequency range, the mode spectrum becomes denser and denser towards the asymptote at f_1, showing a particular 'clustering' phenomenon. It should also be emphasized that the slab's width w affects significantly the location of the modal curves mainly in zones 1 and 3: when w decreases these curves shift towards higher frequencies, and vice-versa.

A further peculiarity is related to certain ranges where power is carried in ferrite and air in opposite directions. The regions where such travel of power is opposite have been evaluated analytically and it has been found that they are internal to the zones 1 and 2: such regions are reported in Fig. 2 (dotted areas).

In zone 3, as frequency increases, modes tend to assume a quasi-isotropic behavior and the nonreciprocity effects are reduced ('modified dielectric' modes). Nevertheless, it should be noted that the first guided mode of region 3, $TE_{1+}^{(3)}$, may present anomalous behaviors depending on the slab's width w. In particular, as w increases, a certain frequency range can be found where this mode has a forward/backward/forward pattern with an evanescent field instead of a stationary one, as should be typical for the other modes in zone 3.

Other important peculiarities, particularly involving the radiation effects, may be found when the eigensolutions of Eq. (1) are calculated on the complex plane.

2.3. *Complex modes in the lossless structure*

As regards the improper real ($\alpha_z=0$, $\alpha_{xo}<0$) and the complex (proper and improper) solutions, it should be pointed out that, due to nonreciprocity, pairs of complex conjugate solutions exist only of the type $k_z=\beta_z\pm j\alpha_z$ (the solutions with opposite sign of β_z do not exist). From a physical viewpoint, such solutions for open structures may describe radiation phenomena, while for closed structures they can take into account only reactive phenomena [4] (specific analysis on this topic will be provided in Sect. 3).

In general, each mode presents both a proper (spectral) and an improper (nonspectral) branch. Moreover, as frequency decreases, complex solutions occur for most of the modes. Due to space limits, a complete analysis of these complex solutions cannot be described here.

We will just briefly refer to the interesting zone 3, showing in Fig. 3 a representative plot of β_z/k_o vs. f for the first three complex modes. In addition to proper real guided waves (solid lines) and relevant nonspectral real waves (dotted lines), we should notice that complex waves of both improper (dashed-dotted lines) and proper (thin lines) type can be found. For the complex solutions of the higher-order modes, again the nonreciprocal effects tend to be reduced and quite standard patterns may be noticed (e.g., transition regions, physical and nonphysical ranges for leaky waves, etc.). For the lower-order complex modes, unconventional nonreciprocal phenomena may be found.

The physical meaning of all these solutions has to be evaluated carefully using suitable procedures, as considered in Sect. 3. However, it is important to note that material losses can deeply change in complicated fashions the modal propagation behaviors. Basic qualitative and quantitative information on this subject is analyzed next.

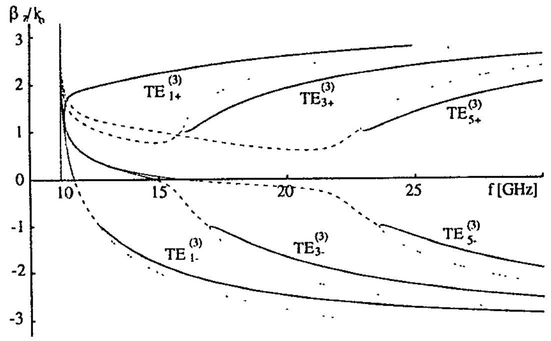

Fig. 3 - Dispersion behavior β_z/k_o vs. f in zone 3 for the first three couples of nonreciprocal modes: proper or guided waves (solid), improper real waves (dotted) and complex waves (thin, if transversely attenuated; dashed/dotted, if transversely increasing).

2.4. *Effects of losses in ferrite*

The effect of losses in the ferrite medium has been taken into account through the classical Landau-Lifshitz model [2], by introducing in the permeability tensor elements a damping factor α. Thus, the physical parameters show no more ideal dispersion behaviors (e.g., infinite values of μ_1, μ_2, and μ_e cannot be reached anymore).

The evaluation of the modal dispersion behaviors in the lossy case (phase and attenuation constants) requires particular attention and has been achieved with specific numerical procedures. The main effects related to the presence of ferrite losses are worthy of a brief discussion.

It can be noted that losses strongly affect the dispersive modal properties particularly in the proximity of the asymptotes f_1 and f_2. Thus, e.g., the phenomenon of mode clustering in the upper part of zone 1 practically disappears (the infinite number of modes is drastically reduced, even though very limited damping factors are considered). The above-described anomalous pattern of the $TE_{1+}^{(3)}$ mode is particularly sensitive to dissipation as well.

Therefore, the amount of loss (in the limits where the considered model is still valid) can change in an involved way the global propagation patterns in connection with the choice of other involved parameters as well (e.g., w). In general, with the introduction of losses, dispersion curves of modes lying in different regions are variously joined across the asymptotes. An example is described by full-wave dispersion behaviors of Fig. 4 for fixed α and w. We should notice that the backward portions of the lossy-case β_z-curves close to f_1 and f_2 are usually related to strong reactive phenomena (high α_z-values), and generally correspond to modifications of complex solutions for the lossless case.

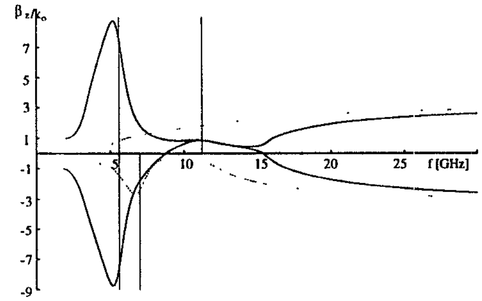

Fig. 4 - Effect of losses on the propagation for a fixed value of the damping factor (α=*0.1*, width and other parameters as in Fig. 1): behavior of β_z/k_o vs. f. In this case (see also the reference lossless curves), the $TE_1^{(1)}$ appears practically joined to the $TE_3^{(3)}$ mode (solid curve), while the $TE_1^{(2)}$ is joined to the $TE_1^{(3)}$ mode (dashed curve).

3. Radiation and leakage effects

3.1. *Spectral representation of the field due to a source*

The previously-described proper and improper solutions for the grounded ferrite slab can be studied as regards their physical meaning and excitation by considering the fields generated by a proper current source. In our problem, we are interested in the TE-mode Green's function for a line source excitation in the magnetization direction, of the type $J_y=\delta(x-x')\delta(z-z')$. The relevant derivation is easily achievable by using the transmission line approach presented in [6], that allows us to use the well-known techniques of spectral-domain approach (SDA).

After analytical manipulations, we can reach the reference spectral expression for the electric field in a point (x,z) in the air, produced by a y-directed and y-independent line source, located at (x',z') inside the ferrite slab:

$$E_y(x,z)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{\dfrac{\sin(k_{xf}x')}{\sin(k_{xf}w)}\,e^{-jk_{xo}(x-w)}\,e^{-jk_z(z-z')}}{Z\left[Y_o^{TE}+\dfrac{T}{Z}-jY_f^{TE}\cot(k_{xf}w)\right]}\,dk_z \quad (2)$$

where we have introduced the suitable quantities: $Z=j\omega\mu_o\mu_e$, $T=\dfrac{\mu_2}{\mu_1}k_z$, $Y_f^{TE}=k_z/(\omega\mu_o\mu_e)$, $Y_o^{TE}=k_z/(\omega\mu_o)$.

This expression has been evaluated with numerical techniques (both rigorous and approximated), and information on radiation effects in our structure has been obtained to be compared with possible leaky-wave contributions.

3.2. *Steepest-descent analysis*

As already pointed out, our structure can support both spectral (proper) and nonspectral (improper) complex modes. Typically, the spectral solutions are not excited singularly and cannot represent radiated power, describing only reactive field in the neighborhood of the source.

The improper leaky modes, conversely, can play an important role in explaining the radiation phenomena.

As previously said, due to the anisotropy of ferrite, complex solutions are still present in complex-conjugate pairs, but, unlike the isotropic structures, the opposite solution and its conjugate are not present. This characteristic makes our structure very interesting if analyzed on the steepest-descent (SD) plane [1]. Thus, it can be noted that for some frequency and parameter ranges, only one leaky pole can be captured when the SD path is moved, by changing the observation angle from the reverse to the direct end-fire, while in the isotropic case there is at least a pair of symmetrical poles that are crossed. In other words, our radiation diagram could be strongly asymmetric with respect to the source location plane at $z=z'$, and a prevalently 'mono-directional' radiation may be related to the capture of a leaky pole that has no correspondent one in almost symmetric positions on the SD plane. A quantitative proof of these assertions is now presented.

3.3. *Radiation field and leaky-wave contributions*

The anisotropic nature of the substrate accounts for a radiation pattern that can be extremely variable as the frequency or the magnetization is changed and is always asymmetric with respect to the source plane location at $z=z'$. However, as already noted, for higher frequencies, where the modal solutions present similarities with the isotropic case, such an asymmetry in the radiation pattern should be reduced. On the contrary, we can expect a strong distorsion effect of the radiation pattern in regions where the nonreciprocal nature of the structure is more evident.

For instance, we can focus our attention on two different frequency regions of zone 3. First, we have selected a region that corresponds to high values of frequency (e.g., around $f=22$ *GHz*, in the case under analysis), where nonreciprocity is weak (the reverse and direct leaky solutions are located in almost symmetric positions). The field pattern is only slightly asymmetric and similar to a cylindrical wave, as calculated via Eq. (2).

Then, we have considered a region where the anisotropic effects are stronger (e.g., around $f=11.8$ *GHz*), close to the asymptote at f_2. In such a frequency range, it results that only the reverse $TE_{1-}^{(3)}$ leaky mode is physical.

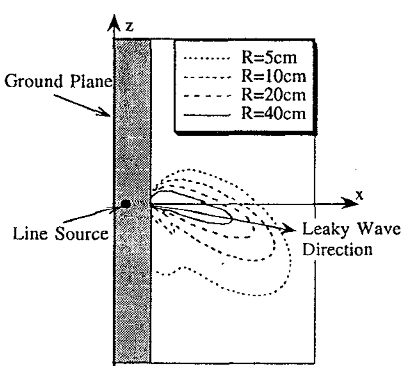

Fig. 5 - Radiation pattern due to a y-directed line source in grounded ferrite slab: 'mono-directional' radiated electric-field magnitude calculated on cylindrical surfaces of increasing radius R, as shown in the legend, at $f=11.8$ *GHz*.

In this case the radiation should heavily be dominated by the leaky-wave contribution and the far-field pattern should appear close to an ideal leaky-wave pattern. In fact, power propagates mostly in one direction that approaches the ideal leaky-pole direction as the distance increases and reaches the far-field region, as is shown in Fig. 5, where the radiated field has been computed through Eq. (2) on cylindrical surfaces of increasing radius R.

It is important to note that the agreement between the far field due to the leaky-wave contribution and the total radiated field for large distances has been proven highly remarkable.

4. Conclusion

A basic class of nonreciprocal gyrotropic waveguides, investigated with complete and accurate procedures, has shown a good number of novel and important properties, concerning both propagation and radiation.

Starting from the lossless case, we have described a variety of basic modal properties, some of which are quite peculiar of nonreciprocal dispersive structures. Then, we have quantified how the material losses can seriously affect the modal distribution, changing dramatically some anomalous characteristic phenomena.

After a full-wave analysis of complex solutions, it has been possible to investigate the contributions given to the radiation phenomena through suitable spectral representations. Some previously-unexplored radiation phenomena have been demonstrated to occur, which can be related to the excitation of disregarded leaky modes.

The results of such analysis provide interesting information of which phenomena can affect the practical properties of such structures. Some of the discovered features may suitably be employed to accurately design types of nonreciprocal passive components and novel solutions of leaky-wave antennas.

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